14.5

13. If $z = f(x, y)$, where f is differentiable, and

$$x = g(t) \qquad y = h(t)$$
$$g(3) = 2 \qquad h(3) = 7$$
$$g'(3) = 5 \qquad h'(3) = -4$$
$$f_x(2, 7) = 6 \qquad f_y(2, 7) = -8$$

find dz/dt when $t = 3$.

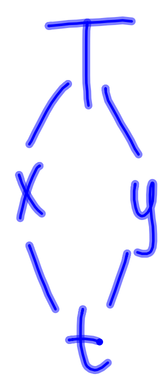
$$\begin{tikzcd} & z \arrow[dl, dash] \arrow[dr, dash] & \\ x \arrow[dr, dash] & & y \arrow[dl, dash] \\ & t & \end{tikzcd}$$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$
$$= 6 \cdot 5 - 8 \cdot (-4)$$
$$= 30 + 32 = 62$$

35. The temperature at a point (x, y) is $T(x, y)$, measured in degrees Celsius. A bug crawls so that its position after t seconds is given by $x = \sqrt{1 + t}$, $y = 2 + \frac{1}{3}t$, where x and y are measured in centimeters. The temperature function satisfies $T_x(2, 3) = 4$ and $T_y(2, 3) = 3$. How fast is the temperature rising on the bug's path after 3 seconds? dt

dT

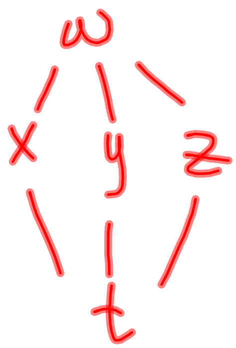
$$\left.\frac{dT}{dt}\right|_{t=3} = \frac{\partial T}{\partial x}\frac{dx}{dt} + \frac{\partial T}{\partial y}\cdot\frac{dy}{dt} \qquad \frac{dy}{dt} = \frac{1}{3}$$



$$\frac{dx}{dt} = \frac{1}{2\sqrt{1+t}} \qquad = 4\left(\frac{1}{4}\right) + 3\left(\frac{1}{3}\right) = 2$$

$2\ {}^\circ C/\text{sec}.$

$\boxed{5.}$ $w = xe^{y/z}, \quad x = t^2, \quad y = 1 - t, \quad z = 1 + 2t$



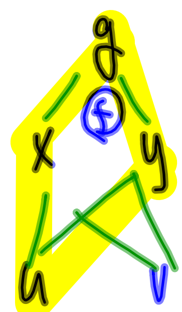
$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{dw}{dz}\frac{dz}{dt}$$

$$= e^{y/z}\cdot 2t + \frac{x}{z}e^{y/z}(-1) - \frac{xy}{z^2}e^{y/z}(2)$$

15. Suppose f is a differentiable function of x and y, and $g(u, v) = f(e^u + \sin v, \boxed{e^u + \cos v})$. Use the table of values to calculate $g_u(0, 0)$ and $g_v(0, 0)$.

$f(x, \boxed{y})$

	f	g	f_x	f_y
(0, 0)	3	6	4	8
(1, 2)	6	3	2	5



$g_u = f_x \cdot x_u + f_y \cdot y_u$

$= 2 \cdot 1 + 5 \cdot 1$

$= 7$

$x_u = e^u$

$y_u = e^u$

33. $x - z = \arctan(yz)$

$$\frac{\partial z}{\partial x} = \frac{-\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}$$

$$F(x,y,z) = x - z - \arctan(yz)$$

$$= \frac{-1}{-1 + \frac{-y}{1+(yz)^2}}$$